(1)

Solve the following LPP using Dynamic programming technique

Maximize $Z = 10x_1 + 30x_2$, subject to the constraints

$3x_1 + 6x_2 \le 168$ and $12x_2 \le 240$, x_1 and $x_2 \ge 0$.

Solution This LP problem can be considered as a two stage, two state problem.

Starting with the second stage backward, the procedure as follows.

The optimal value of $f_2(b_1, b_2) = \underset{0 \le x_2 \le b_2}{\text{Max}} \{30x_2\}$

Here $b_1 = 168$, $b_2 = 240$.

The feasible value of x_2 is a non-negative value that satisfies all the given constraints

$6x_2 \le 168$ and $12x_2 \le 240$

$x_2 \le 28$ $\quad x_2 \le \frac{240}{12}$

$x_2 \le 20$

$b = \min\{28, 20\} = 20$

$$f_2(b_1, b_2) = \underset{0 \le x_2 \le 20}{\max} \{30x_2\} = 30 \underset{0 \le x_2 \le 20}{\text{Min}} \{28 - \tfrac{1}{2}x_1, \cancel{\tfrac{x_2}{2}} 20\}$$

$$\therefore x_2 = \underset{0 \le x_2 \le 20}{\text{Min}} \{28 - \tfrac{1}{2}x_1, 20\}$$

Proceeding backward to stage 1, the recursive relation for optimization can be expressed as,

$$f_1(b_1, b_2) = \underset{0 \le x_1 \le b}{\text{Max}} \{10x_1 + 30 \text{ Min}\{28 - \tfrac{1}{2}x_1, 20\}\}$$

Maximization of variable x_1 satisfying the condition

$3x_1 \le 168 \quad x_1 \le 56$

$b = 56$ ②

$$\therefore f_1(b_1, b_2) = \underset{0 \le x_1 \le 56}{\text{Max}} \left\{ 10x_1 + 30 \,\text{Min}\left\{28 - \tfrac{1}{2}x_1 \,,\, 20\right\} \right\}$$

Now

$\text{Min}\left\{28 - \tfrac{1}{2}x_1 \,,\, 20\right\} \quad 0 \le x_1 \le 56$

when $x_1 = 0$	20
$x_1 = 10$	20
$x_1 = 15$	20
$x_1 = 16$	20
$16 \le x_1 \le 56$	$28 - \tfrac{1}{2}x_1$

~~Min 0~~

~~when $x_1 = 56$ the optimal Value of x_2 is~~

$\therefore$ ~~$x_1 = 56$~~

~~$x_2 = \text{Min}\{0, 20\} = 20$~~

$$f_1(b_1, b_2) = \text{Max} \begin{bmatrix} 10x_1 + 30 \min\left(28 - \tfrac{1}{2}x_1 \,,\, 20\right) & 0 \le x_1 \le 16 \\ 10x_1 + 30 \min\left(28 - \tfrac{1}{2}x_1 \,,\, 20\right) & 16 \le x_1 \le 56 \end{bmatrix}$$

$$= \text{Max} \begin{bmatrix} 10x_1 + 30 \times 20 & \\ 10x_1 + 30\left(28 - \tfrac{1}{2}x_1\right) & 16 < x_1 \le 56 \end{bmatrix}$$

$$= \text{Max} \begin{bmatrix} 10x_1 + 600 & 0 \le x_1 \le 16 \\ 10x_1 + 840 - 15x_1 & 16 < x_1 \le 56 \end{bmatrix}$$

$$= \text{Max} \begin{bmatrix} 10x_1 + 600 & 0 \le x_1 \le 16 \\ 840 - 5x_1 & 16 < x_1 \le 56 \end{bmatrix}$$

when $x_1 = 16 \quad \text{Max}[760, 760]$

$\therefore x_1 = 16$ is optimal Value

when $x_1 = 16$

$$x_2 = \text{Min}\{28-8, 20\} = 20$$

$\therefore$ optimal solution is

$$x_1 = 16, \quad x_2 = 20$$

Maximum of $Z = 10x_1 + 30x_2$

$$Z = 10(16) + 30(20)$$

$$= 160 + 600$$

$$\text{Max } Z = 760$$

$\therefore$ optimal solution is $x_1 = 16$, $x_2 = 20$

$$\text{Max } Z = 760.$$